Note Title 7/11/2010

# STT 200-202 7-12-10

PROBABILITY RULES — ADDITION, MULTIPLICATION — INDEPENDENT CASE / GENERAL

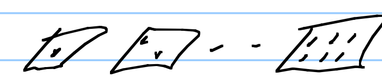
CONDITIONAL PROBABILITY

BINOMIAL

BINOMIAL APPROXIMATION BY NORMAL DISTRIBUTION

## PROBABILITY

CLASSICAL ELEMENTARY OUTCOMES. eg DIE



or 1 2 ... 6

ALL EQUALLY LIKELY pr 1/6 ... 1/6

PICTURE

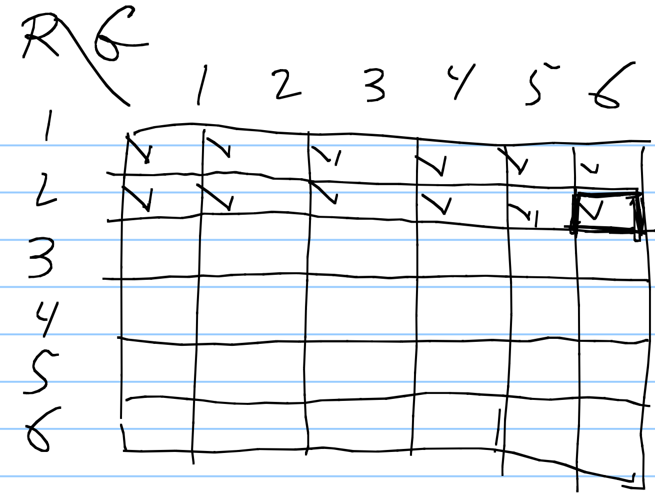
1 2 3
4 5 6

OUTCOME SPACE (SAMPLE SPACE)

$p(\text{less than } 3) = 2/6$

$P(2) = 1/6 \quad p(1 \text{ or } 2) = 2/6 =$ #FAVORABLE / #TOTAL

TWO DICE RED DIE ; GREEN DIE



$P(r = 2 \text{ and } g = 6)$

$P(r < 3) = \frac{12}{36} = \frac{2}{6}$

JACK + JILL $\boxed{\$1, \$1, \$5}$

JACK DRAWS A BILL

THEN JILL DRAWS A BILL FROM THE 2 REMAINING

? $P(\text{JACK } \$5) \overset{\text{INTUIT}}{=} 1/3$

? $P(\text{JILL } \$5) =$

DRAWING BILLS NOT \$ AMOUNTS $\boxed{\$1_a \ 1_b \ 5_c}$

$\frac{1}{3}$ $\frac{1}{2}$ !!

$\frac{1}{8}$ $\frac{1}{2}$ $\frac{1}{6}$ $\frac{1}{3}$

0 0 6 5

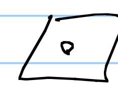
| Pr | JACK | JILL | JACK $5 = ✓ | JILL $5 * |
|---|---|---|---|---|
| 1/6 | a | b | | |
| 1/6 | a | c | | * |
| 1/6 | b | a | | |
| 1/6 | b | c | | * |
| 1/6 | c | a | ✓ | |
| 1/6 | c | b | ✓ | |
| 1 | | | | |

6 ELEMENTARY OUTCOMES

P(JILL $5) = 2/6 = 1/3

P(JACK $5) = 2/6 = 1/3

REVISING PROBABILITY BASED ON PARTIAL INFORMATION ABOUT THE OUTCOME.

eg TOSS DIE, "KNOW IT WAS NOT  "

REVISED P(2)

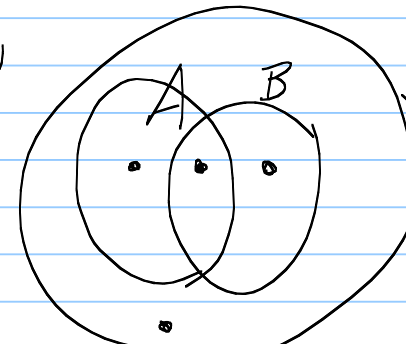
| ~~1~~ | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| ~~1/6~~ | 1/6 | 1/6 | 1/6 | 1/6 | 1/6 |

EACH OF 2 → 6 REMAIN EQUALLY PROBABLE.

SO $\boxed{P(\text{DIE IS 2} \mid \text{DIE IS NOT 1})}$ MUST BE $\frac{1}{5}$.

IF (GIVEN)

EVENT A
EVENT B
SAMPLE SPACE




GEN'L. $P(\overset{\text{male}}{A}) = 4/19$ $\quad P(A^c \text{ or } \bar{A}) = \frac{15}{19}$ 15 WOMEN

$= 1 - P(A)$

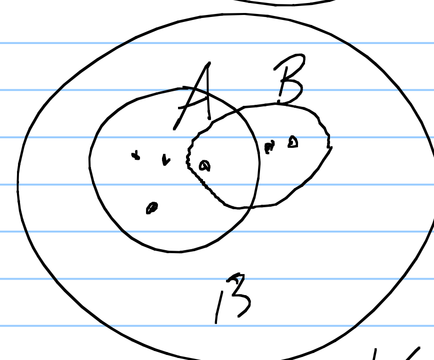
eg A = MALE, B = ON CAMPUS

$P(A \cap B) = P(A \text{ and } B) = \frac{1}{19}$ male on campus

INTERSECTION

$P(A \text{ or } B) = P(A \cup B) = 6/19$

UNION




TOTAL 19

ADDITION RULE:

$P(A \cup B) = P(A) + P(B) - P(A \cap B) = 6/19$

or $\quad 4/19 \quad 3/19 \quad 1/19$ and

Venn

FOR EXAMPLE WEATHER FORECAST

SAYS $P(\overset{A}{\text{RAIN SAT}}) = .36$

" ALSO $P(\underset{B}{\text{RAIN SUN}}) = .17$ FOR EXAMPLE

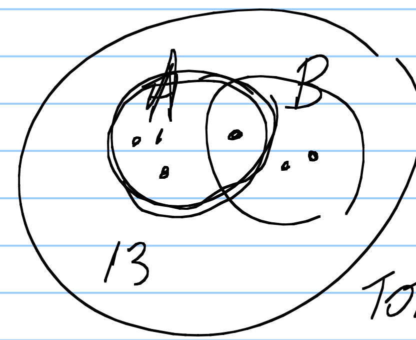
$\Rightarrow P(\text{RAINS SAT OR SUN}) = \underset{\text{RAIN SAT}}{P(A)} + \underset{\text{RAIN SUN}}{P(B)} - \underset{\text{RAIN BOTH DAYS}}{P(A \cap B)}$

DON'T KNOW THIS!

ATTACK $P(A \cap B) = \dfrac{\#(A \cap B)}{\#\text{TOTAL}}$

$= \dfrac{\#A}{\#\text{TOTAL}} \; \dfrac{\#(A \cap B)}{\#A}$

FORCED THIS ON YOU $= P(A) \; P(B \,|_{\text{IF (GIVEN)}}\, A)$




TOTAL 19

CLAIM: $P(A \underset{\text{and}}{\cap} B) \overset{\text{WANT}}{=} P(A) \; P(B \,|_{\text{IF}}\, A)$

SAME AS SAYING $\frac{P(A \underset{\text{and}}{\cap} B)}{P(A)}$ MUST BE $= P(B \mid_{IF} A)$

RETURN TO JACK + JILL. $\boxed{1_a \ 1_b \ 5_c}$

$P(\text{JACK } 1) = 2/3$

INTUIT

$P(\text{JILL } 5 \mid_{IF} \text{JACK } 1) = \frac{1}{2}$ B-CAUSE JILL IS DRAWING FROM $\boxed{1 \ 5}$

ABOVE RULE SAYS $P(\text{JACK } 1 \text{ and JILL } 5)$

$$= P(\text{JACK } 1)\, P(\text{JILL } 5 \mid_{IF} \text{JACK } 1) = \frac{2}{3}\,\frac{1}{2} = \frac{1}{3}$$

JACK and JILL 5 IS THE ONLY WAY JILL GETS 5.

DRAW BALLS WITHOUT REPLACEMENT:

$\boxed{4R \quad 7G \quad 2Y}$

NOTATION: $R_1$ MEANS RED DRAWN ON FIRST DRAW

$$\left.\begin{aligned} P(R_1) &= 4/13 \\ P(R_2 \mid_{\text{IF}} R_1) &= 3/12 \end{aligned}\right\} \text{ SO } \underbrace{P(R_1 \text{ and } R_2)}_{} = \tfrac{4}{13}\,\tfrac{3}{12}$$

$$P(R_1^c) = 1 - 4/13 = 9/13$$

$$\underbrace{P(R_1^c \text{ and } R_2) = \tfrac{9}{13}\,\tfrac{4}{12}}_{}$$

DO NOT OVERLAP

NOTHING TO SUBTRACT IN ADDN RULE

SO ADDN RULE $\Rightarrow$ $P(R_2) = P(\underbrace{R_1^c \text{ and } R_2}_{} \text{ or } \underbrace{R_1 \text{ and } R_2}_{})$

$$= \tfrac{9}{13}\,\tfrac{4}{12} \underset{\text{or}}{+} \tfrac{4}{13}\,\tfrac{3}{12} - 0$$

$$= \frac{36+12}{13 \cdot 12} = \frac{48}{13 \cdot 12} = 4/13.$$ SAME AS $P(R_1)$ !!

BINOMIAL: a. SEQUENCE OF TRIALS eg TOSS COIN REPEATEDLY

BERNOULLI TRIALS

b. EACH TRIAL HAS ONE OF TWO OUTCOMES eg H or T. "SUCCESS" "FAILURE"

c. P("SUCCESS") SAME FOR EACH TRIAL

d. TRIALS ARE STATISTICALLY <u>INDEPENDENT</u>

MEANS e.g. $P(H_6 \text{ and } T_9 \mid_{IF} \cancel{H_1 \text{ and } T_2}) = P(H_6 \text{ and } T_9)$ TELLS YOU NOTHING

MULT RULE $= P(H_6)\, P(T_9 \mid_{IF} \cancel{H_6}) = \frac{1}{2} \cdot \frac{1}{2} = 1/4$

ANOTHER EXAMPLE OF BERNOULLI TRIALS:

(1) PARTS (CASTINGS) FROM PRODUCTION RANDOMLY SAMPLED.

"SUCCESS" DEFECTIVE

"FAILURE" NOT DEFECTIVE.

SUPPOSE ONLY 1.5% ARE DEFECTIVE (PROCESS RATE)

CAP P $\quad$ $p = 0.015$ SCALE $0 \to 1$. $\quad$ $P(S_2 \mid S_1) \overset{\text{INDEP}}{=} P(S_2)$ IF

$$P(S_1\ S_2\ F_3\ F_4\ S_5) \overset{\text{INDEP OF TRIALS}}{=} (0.015)(0.015)(1-.015)(1-.015)(0.015) \quad \text{IF INDEPENDENT}$$

$S_1$: DEF $\quad$ $F_3$: NOT DEF

$$= (0.015)^3 (1-0.015)^{5-3=2}$$

? P( GET TOTAL OF 3 "5" IN 5 TRIES) ← FIX THIS

$$= \boxed{\begin{array}{l}\text{\# OF ARRANGEMENTS} \\ C(5,3)\end{array}} (.015)^3 (1-.015)^{5-3=2}$$

$$= 5!/(2!\,3!)\ .015^3 (1-.015)^{5-3=2}$$

$$\frac{5! = 5 \cdot 4 \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}}{2\ 1\ \ \cancel{3}\ \cancel{2}\ 1} = 10$$

555FF $p^3(1-p)^2$

55~~F~~5F $p^3(1-p)^2$

55FF5

⋮

FF555 $p^3(1-p)^2$